Wurzelgleichungen

Dokumentnummer: DX1752
Fachgebiet: Gleichungen, Potenzen und Wurzeln, binomische Formeln
Einsatz: 2HAK (erstes Lernjahr)
Quelle: Steiner Weilharter, BD1, HPT
Didaktischer Hinweis: für Wurzelgleichungen ist solve() nicht geeignet

1 Aufgaben und Lösungen

Figure 1:

818. a) $5\sqrt{3x+1}=3\sqrt{5x+25}$ **b)** $4\sqrt{4x+1}=3\sqrt{7x+2}$

```
(%i1) g818a:5*sqrt(3*x+1)=3*sqrt(5*x+25);
      g818b:4*sqrt(4*x+1)=3*sqrt(7*x+2);
```

(%o1) $5\sqrt{3x+1}=3\sqrt{5x+25}$

(%o2) $4\sqrt{4x+1}=3\sqrt{7x+2}$

```
(%i3) to_poly_solve(g818a,x);
```

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(%o3) $\%union\left(\left[x=\frac{20}{3}\right]\right)$

```
(%i4) to_poly_solve(g818b,x);
```

(%o4) $\%union([x=2])$

Figure 2:

819. a) $5\sqrt{x}-1=7\sqrt{x}-5$ **b)** $7\sqrt{x}+9=6(3\sqrt{x}-4)$

```
(%i5) g819a:5*sqrt(x)-1=7*sqrt(x)-5;
      g819b:7*sqrt(x)+9=6*(3*sqrt(x)-4);
```

(%o5) $5\sqrt{x}-1=7\sqrt{x}-5$

(%o6) $7\sqrt{x}+9=6(3\sqrt{x}-4)$

```
(%i7) to_poly_solve(g819a,x);
```

(%o7) $\%union([x=4])$

```
(%i8) to_poly_solve(g819b,x);
```

(%o8) $\%union([x=9])$

Figure 3:

820. a) $\sqrt{x+3}-\sqrt{x}=1$ **b)** $\sqrt{x+14}+\sqrt{x+7}=7$

```
(%i9) g820a:sqrt(x+3)-sqrt(x)=1;
      g820b:sqrt(x+14)+sqrt(x+7)=7;
```

(%o9) $\sqrt{x+3}-\sqrt{x}=1$

(%o10) $\sqrt{x+14}+\sqrt{x+7}=7$

```
(%i11) to_poly_solve(g820a,x);
(%o11) %union([x=1])
```

```
(%i12) to_poly_solve(g820b,x);
(%o12) %union([x=2])
```

Figure 4:

821. a) $\sqrt{x+9}-\sqrt{x-4}=1$ **b)** $\sqrt{x-5}+\sqrt{x+16}=7$

```
(%i13) g821a:sqrt(x+9)-sqrt(x-4)=1;
       g821b:sqrt(x-5)+sqrt(x+16)=7;
```

(%o13) $\sqrt{x+9}-\sqrt{x-4}=1$

(%o14) $\sqrt{x+16}+\sqrt{x-5}=7$

```
(%i15) to_poly_solve(g821a,x);
(%o15) %union([x=40])
```

```
(%i16) to_poly_solve(g821b,x);
(%o16) %union([x=9])
```

Figure 5:

822. a) $\sqrt{5x-9}-\sqrt{5x+11}=-2$ **b)** $\sqrt{5x+11}-\sqrt{5x+4}=1$

```
(%i17) g822a:sqrt(5*x-9)-sqrt(5*x+11)=-2;
       g822b:sqrt(5*x+11)-sqrt(5*x+4)=1;
```

(%o17) $\sqrt{5\,x-9}-\sqrt{5\,x+11}=-2$

(%o18) $\sqrt{5\,x+11}-\sqrt{5\,x+4}=1$

```
(%i19) to_poly_solve(g822a,x);
(%o19) %union([x=5])
```

```
(%i20) to_poly_solve(g822b,x);
(%o20) %union([x=1])
```

Figure 6:

823. a) $\sqrt{3x+4}=17-\sqrt{3x+123}$ **b)** $\sqrt{4x-15}=2\sqrt{x-8}+1$

```
(%i21) g823a:sqrt(3*x+4)=17-sqrt(3*x+123);
       g823b:sqrt(4*x-15)=2*sqrt(x-8)+1;
```

(%o21) $\sqrt{3\,x+4}=17-\sqrt{3\,x+123}$

(%o22) $\sqrt{4\,x-15}=2\,\sqrt{x-8}+1$

```
(%i23) to_poly_solve(g823a,x);
(%o23) %union([x=7])
```

```
(%i24) to_poly_solve(g823b,x);
(%o24) %union([x=24])
```

Figure 7:

824. a) $\sqrt{x+4}+\sqrt{x+11}=\sqrt{4x+29}$ **b)** $\sqrt{x+1}+\sqrt{4x+4}=\sqrt{9x+9}$

```
(%i25) g824a:sqrt(x+4)+sqrt(x+11)=sqrt(4*x+29);
       g824b:sqrt(x+1)+sqrt(4*x+4)=sqrt(9*x+9);
```

(%o25) $\sqrt{x+11}+\sqrt{x+4}=\sqrt{4\,x+29}$

(%o26) $\sqrt{4\,x+4}+\sqrt{x+1}=\sqrt{9\,x+9}$

```
(%i27) to_poly_solve(g824a,x);
```

(%o27) %union([$x=5$])

```
(%i28) to_poly_solve(g824b,x);
```

(%o28) %union

$\left(\%\mathrm{if}\left(\left(-\frac{\pi}{2}<\mathrm{parg}(\%c70)\right)\ \%and\ \left(\mathrm{parg}(\%c70)<=\frac{\pi}{2}\right),[x=\%c70^2-1],\%\mathrm{union}(\,)\right),[x=-1]\right)$

Wir müssen 824b näher untersuchen

```
(%i29) g824b;
```

(%o29) $\sqrt{4\,x+4}+\sqrt{x+1}=\sqrt{9\,x+9}$

```
(%i30) g1:lhs(g824b)**2=rhs(g824b)**2,expand;
```

(%o30) $2\sqrt{x+1}\,\sqrt{4\,x+4}+5\,x+5=9\,x+9$

```
(%i31) g2:g1-5*x-5;
```

(%o31) $2\sqrt{x+1}\,\sqrt{4\,x+4}=4\,x+4$

```
(%i32) g3:lhs(g2)**2=rhs(g2)**2,expand;
```

(%o32) $16\,x^2+32\,x+16=16\,x^2+32\,x+16$

```
(%i33) g4:g3-lhs(g3);
```

(%o33) $0=0$

Die Gleichung hat unendlich viele Lösungen?
(Jedenfalls im Bereich der komplexen Zahlen).

```
(%i34) g824b,x=9728;
```

(%o34) $9\sqrt{1081}=9\sqrt{1081}$

Eine exemplarische Probe.

Figure 8:

825. a) $\sqrt{4x+1}-\sqrt{x+3}=\sqrt{x-2}$ **b)** $\sqrt{x+3}+\sqrt{x+8}=\sqrt{4x+21}$

```
(%i35) g825a:sqrt(4*x+1)-sqrt(x+3)=sqrt(x-2);
       g825b:sqrt(x+3)+sqrt(x+8)=sqrt(4*x+21);
```

(%o35) $\sqrt{4\,x+1}-\sqrt{x+3}=\sqrt{x-2}$

(%o36) $\sqrt{x+8}+\sqrt{x+3}=\sqrt{4\,x+21}$

```
(%i37) to_poly_solve(g825a,x);
```
(%o37) %union([x=6])

```
(%i38) to_poly_solve(g825b,x);
```
(%o38) %union([x=1])

Figure 9:

826. a) $\sqrt{x+15}+\sqrt{x+3}=2\sqrt{x+8}$ **b)** $\sqrt{x+21}+\sqrt{x+5}=2\sqrt{x+12}$

```
(%i39) g826a:sqrt(x+15)+sqrt(x+3)=2*sqrt(x+8);
       g826b:sqrt(x+21)+sqrt(x+5)=2*sqrt(x+12);
```
(%o39) $\sqrt{x+15}+\sqrt{x+3}=2\sqrt{x+8}$

(%o40) $\sqrt{x+21}+\sqrt{x+5}=2\sqrt{x+12}$

```
(%i41) to_poly_solve(g826a,x);
```
(%o41) %union([x=1])

```
(%i42) to_poly_solve(g825b,x);
```
(%o42) %union([x=1])

Figure 10:

827. a) $3\sqrt{x+3}+\sqrt{x+6}=\sqrt{4x+33}$ **b)** $2\sqrt{x+3}+3\sqrt{x+8}=\sqrt{25x+144}$

```
(%i43) g827a:3*sqrt(x+3)+sqrt(x+6)=sqrt(4*x+33);
       g827b:2*sqrt(x+3)+3*sqrt(x+8)=sqrt(25*x+144);
```
(%o43) $\sqrt{x+6}+3\sqrt{x+3}=\sqrt{4x+33}$

(%o44) $3\sqrt{x+8}+2\sqrt{x+3}=\sqrt{25x+144}$

```
(%i45) to_poly_solve(g827a,x);
```
(%o45) %union([x=-2])

```
(%i46) to_poly_solve(g827b,x);
```
(%o46) %union([x=1])

Figure 11:

828. a) $\sqrt{x+12}-\sqrt{x-3}=\sqrt{x+32}-\sqrt{x+5}$ **b)** $\sqrt{x+12}+\sqrt{x-3}=\sqrt{x+32}-\sqrt{5+x}$

```
(%i47) g828a:sqrt(x+12)-sqrt(x-3)=sqrt(x+32)-sqrt(x+5);
       g828b:sqrt(x+12)+sqrt(x-3)=sqrt(x+32)-sqrt(5-x);
```
(%o47) $\sqrt{x+12}-\sqrt{x-3}=\sqrt{x+32}-\sqrt{x+5}$

(%o48) $\sqrt{x+12}+\sqrt{x-3}=\sqrt{x+32}-\sqrt{5-x}$

```
(%i49) to_poly_solve(g828a,x);
```
(%o49) %union([x=4])

```
(%i50) to_poly_solve(g828b,x);
```
(%o50) %union([x=4] , [x=4.164361269324655])

Das wäre wieder ein Ergebnis zum Nachrechnen.

(%i51) `g828b;`

(%o51) $\sqrt{x+12}+\sqrt{x-3}=\sqrt{x+32}-\sqrt{5-x}$

(%i52) `g1:lhs(g828b)**2=rhs(g828b)**2,expand;`

(%o52) $2\sqrt{x-3}\sqrt{x+12}+2x+9=37-2\sqrt{5-x}\sqrt{x+32}$

(%i53) `g2:g1-37;`

(%o53) $2\sqrt{x-3}\sqrt{x+12}+2x-28=-2\sqrt{5-x}\sqrt{x+32}$

(%i54) `g3:g2/2,expand;`

(%o54) $\sqrt{x-3}\sqrt{x+12}+x-14=-\sqrt{5-x}\sqrt{x+32}$

(%i55) `g4:lhs(g3)**2=rhs(g3)**2,expand;`

(%o55) $2\sqrt{x-3}\,x\sqrt{x+12}-28\sqrt{x-3}\sqrt{x+12}+2x^2-19x+160=-x^2-27x+160$

(%i56) `g5:g4-2*x**2+19*x-160,expand;`

(%o56) $2\sqrt{x-3}\,x\sqrt{x+12}-28\sqrt{x-3}\sqrt{x+12}=-3x^2-8x$

(%i57) `g6:lhs(g5)**2=rhs(g5)**2,expand;`

(%o57) $4x^4-76x^3-368x^2+11088x-28224=9x^4+48x^3+64x^2$

Da müssen wir wohl das CAS zu Hilfe nehmen.
Herkömmlich müsste man ein Näherungsverfahren
verwenden.

(%i58) `g7:g6-rhs(g6);`

(%o58) $-5x^4-124x^3-432x^2+11088x-28224=0$

(%i59) `l:realroots(g7),numer;`

(%o59) $[x=4, x=4.164361149072647]$

Figure 12:

829. a) $\sqrt{x+1}+\sqrt{x+6}=\sqrt{x-2}+\sqrt{x+13}$ **b)** $\sqrt{x+6}-\sqrt{x-1}=\sqrt{x+26}-\sqrt{x+15}$

(%i60) `g829a:sqrt(x+1)+sqrt(x+6)=sqrt(x-2)+sqrt(x+13);`
`g829b:sqrt(x+6)+sqrt(x-1)=sqrt(x+26)-sqrt(x+15);`

(%o60) $\sqrt{x+6}+\sqrt{x+1}=\sqrt{x+13}+\sqrt{x-2}$

(%o61) $\sqrt{x+6}+\sqrt{x-1}=\sqrt{x+26}-\sqrt{x+15}$

(%i62) `to_poly_solve(g829a,x);`

(%o62) %union([$x=3$])

(%i63) `to_poly_solve(g829b,x);`

(%o63) %union()

Das sollte man herkömmlich nachrechnen
(warum es keine Lösung gibt).

Figure 13:

830. a) $\sqrt{2x-4}-\sqrt{2x+29}=\sqrt{2x-16}-\sqrt{2x+5}$ **b)** $\sqrt{3x+7}-\sqrt{3x-5}=\sqrt{3x+27}-\sqrt{3x+7}$

```
(%i64) g830a:sqrt(2*x-4)-sqrt(2*x+29)=sqrt(2*x-16)-sqrt(2*x+5);
       g830b:sqrt(3*x+7)-sqrt(3*x-5)=sqrt(3*x+27)-sqrt(3*x-7);
```

(%o64) $\sqrt{2\,x-4}-\sqrt{2\,x+29}=\sqrt{2\,x-16}-\sqrt{2\,x+5}$

(%o65) $\sqrt{3\,x+7}-\sqrt{3\,x-5}=\sqrt{3\,x+27}-\sqrt{3\,x-7}$

```
(%i66) to_poly_solve(g830a,x);
```

(%o66) %union([$x=10$])

```
(%i67) to_poly_solve(g830b,x);
```

(%o67) %union( )

Das sollte man herkömmlich nachrechnen
(warum es keine Lösung gibt).